General Exam Instructions

If you are planning to take the Probability part of your general exam with me, you need to go over my notes that cover both Probability I and II. The latest version is located here:

http://merlot.stat.uconn.edu/~boba/stat6894/probabilityI.pdf

I expect you to memorize all the major definitions and major theorem statements. I will not ask you to reproduce relatively complicated proofs. However, you have to know how to do simple standard tricks. Here are some examples: proving Chebyshev's Inequality, deriving formulas for characteristic function of standard distributions, checking that a sequence of random variables forms a martingale, giving an example of a sequence of random variables that converges in probability but not with probability one.

Here is the list of definitions, concepts, and theorem statements that you absolutely must know. A failure to answer any of these questions automatically means that you failed the exam.

1. Definitions of σ-field.
2. Probability space.
3. Continuity and σ-additiveness of additive measure.
4. Random variable.
5. Definition of independence.
6. Borel-Cantelli lemmas.
7. Expectation of simple random variables.
8. Expectation of random variables.
9. Monotone convergence theorem.
10. Fatou's lemma. Dominated convergence theorem.
11. Chebyshev's inequality.
12. Cauchy-Schwarz's inequality.
13. Jensen's inequality.
14. Radon-Nikodym theorem.
15. Product spaces and Fubini's theorem.
16. Characteristic Functions.
17. Different Types of Convergence: with probability 1, in L_p, in probability, in distribution.
18. Continuity Theorem
19. Week Laws of Large Numbers for iid random variables.
20. Central Limit Theorem for iid random variable.
21. Conditional Expectation.
22. Martingale: Definition.
23. Doob's Optional Stopping Theorem.
24. Doob's Submartingale Inequality.

For example, I can ask you to explain how we define the expectation of non-negative random variable X. Here is the answer.

Since any random variable is a measurable mapping of $(\Omega, \mathcal{F})$ to $(\mathbb{R}, \mathcal{B})$, we can construct a sequence of non-negative simple random variables $\{X_n\}$ such that $X_n(\omega) \uparrow X(\omega)$ for any $\omega \in \Omega$. The expectation of a simple random variable is given by an explicit formula. Then $E(X) = \lim_{n\to\infty} E(X_n)$. This limit exists and does not depend on the approximation of X by X_n.

Additionally, I can ask you to define measurability, to prove the existence of the limit, to define simple random variable etc. I will not ask you to show that the limit is the same for any two approximations.